Hydrodynamic fluctuations at the convective instability*

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The effects of thermal fluctuations on the convective instability are considered. It is shown that the Langevin equations for hydrodynamic fluctuations are equivalent, near the instability, to a model for the crystallization of a fluid in equilibrium. Unlike the usual models, however, the free energy of the present system does not possess terms cubic in the order parameter, and therefore the system undergoes a second-order transition in mean-field theory. The effects of fluctuations on such a model were recently discussed by Brazovskii, who found a first-order transition in three dimensions. A similar argument also leads to a discontinuous transition for the convective model, which behaves two dimensionally for sufficiently large lateral dimensions. The magnitude of the jump is unobservably small, however, because of the weakness of the thermal fluctuations being considered. The relation of the present analysis to the work of Graham and Pleiner is discussed.

I. INTRODUCTION

There has been recent interest[1] in the effects of thermal fluctuations on a fluid near the Rayleigh-Bénard instability.[2] Zaitsev and Shliomis[3] calculated the velocity-correlation function using a linear theory which corresponds to mean-field theory in a phase transition at equilibrium. They found a sharp continuous transition to the convective state, with a diverging "static susceptibility" and critical slowing down of the characteristic frequency. They also calculated exponents describing the correlation length, etc., which are typical mean-field exponents.

One may then pose the following question: How does the nonlinear coupling of fluctuations change the above mean-field result? Does it leave the transition sharp and continuous, but with different values of the exponents? Alternatively, will the transition become sharp but discontinuous, or will the sharp transition be smeared out?

In an early work, Graham[4] has attacked the above problem in a case where the plates between which the fluid is confined are of finite lateral dimension. Graham found that fluctuations smear out the sharp mean-field transition, as in a "zero-dimensional" problem[5,6] with only a finite number of modes, e.g., a laser. The region over which this smearing effect is apparent is extremely narrow, however, because of the smallness of the thermal energy compared to the typical energy of a convective cell. Thus the exact behavior of the fluctuating system is unlikely to be observed experimentally, and the problem under consideration is of theoretical interest only.

In the present work we consider a Bénard system of arbitrarily large lateral dimension and study the effect of fluctuations on the onset of convective motion, using the same statistical starting point as in Refs. 3 and 4. We show that for an infinite system the Bénard problem is equivalent, very near the instability, to a time-dependent relaxational model[7] near equilibrium, which undergoes a "crystallization" from a liquid to a solid. Unlike the usual freezing transition, however, the "free energy" of our model does not contain any terms cubic in the order parameter, owing to the symmetry of the problem. Thus we are dealing with a transition which is second order in mean-field theory, unlike the usual crystallization.[8]

A similar model was recently considered in equilibrium by Brazovskii,[9] in connection with the study of weakly anisotropic antiferromagnets, and certain liquid crystals. The important new feature of this class of models is the high degree of degeneracy of the ordered state, since one is dealing with broken translational *and* rotational symmetries. This is in contrast to the usual antiferromagnetic transition in a crystal, where the periodic structure characterizing the spins is determined, at least in part, by that of the underlying lattice. As we shall see below, the present model does not belong to any of the universality classes[10] previously considered in the study of critical behavior.

In the weak coupling limit, which is certainly applicable to the Bénard case, Brazovskii[9] used perturbation theory to argue that a *first-order* transition would occur in three dimensions, as a result of fluctuations. The model we consider differs from Brazovskii's in the dimensionality, since in our case fluctuations are totally suppressed in the vertical direction. Brazovskii's argument can be repeated in the two-dimensional case, however, and one again finds a *first-order* transition (to a system with "rolls"[2]), as long as the lateral dimension L is not too large [$L/l \ll \lambda^{-2/5}$, where λ is the coupling constant and l the plate separation]. In the case $L/l \gg \lambda^{-2/5}$ in two dimensions, or for $\lambda \gtrsim 1$ in any dimension, the arguments break down, and the behavior of the system is not known. It should be pointed out, moreover, that Brazovskii's treatment is not rigorous, and it is conceivable the sharp jump he obtains will be smeared out slightly when a more precise calculation is performed.

In Sec. II the equivalence of the Rayleigh-Bénard problem and Brazovskii's model is demonstrated, following arguments which are close to those of Refs. 3 and 4. Section III discusses Brazovskii's work, and the modifications which are necessary in two dimensions. In Sec. IV comparison is made with the treatments of Graham[5,6] and Graham and Pleiner,[11] and the reasons for their disagreement with the present results are discussed. The mathematical details of our work are contained in Appendixes A and B.

II. PROOF OF EQUIVALENCE OF CONVECTIVE INSTABILITY TO BRAZOVSKII'S MODEL

Let us begin with the equations of fluid dynamics in the Boussinesq approximation[2] for a fluid bounded by infinite horizontal plates separated by a distance l, and at temperatures T_1 and $T_1 + \Delta T$, respectively,

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \operatorname{grad} \vec{u} = -\operatorname{grad}(p/\rho) + \nu \nabla^2 \vec{u} + g\alpha T \hat{z} , \quad (1)$$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \operatorname{grad} T = \kappa \nabla^2 T , \quad (2)$$

$$\operatorname{div} \vec{u} = 0 , \quad (3)$$

where $\vec{u}(\vec{x}, t)$ is the fluid velocity, $T(\vec{x}, t)$ the temperature, p the pressure, ρ the density, ν the kinematic viscosity, κ the thermal diffusivity, g the acceleration of gravity, and α the thermal expansion. We introduce the variable

$$\theta(\vec{x}, t) = T - T_1 + (\Delta T/l) z , \quad (4)$$

which describes the departure of the temperature from the uniform gradient $\Delta T/l$, and study the coupled equations for θ and $u_z(\vec{x}, t)$. As is well known,[2] these equations have an instability at a critical value of ΔT, given by $R = R_c$, where the Rayleigh number R is defined as

$$R = g\alpha l^3 \Delta T/\nu\kappa . \quad (5)$$

The value of R_c depends on the boundary conditions on the horizontal surfaces.

In order to calculate the fluctuations in hydrodynamic variables *near thermal equilibrium* ($R \to 0$), it is convenient to add a Langevin force[12,3,4] to the right-hand sides of (1) and (2), representing the random effect of molecular noise:

$$\partial_t u_i + u_j \partial_j u_i + \partial_i (p/\rho) - \nu \partial_j \partial_j u_i - g\alpha T \delta_{iz} = \partial_j s_{ij} , \quad (6)$$

$$\partial_t T + u_j \partial_j T - \kappa \partial_j \partial_j T = -\partial_j q_j , \quad (7)$$

where $\partial_t \equiv \partial/\partial t$ and $\partial_j \equiv \partial/\partial x_j$. For $\Delta T \to 0$, the system described by these equations will relax to the usual equilibrium at the given temperature and pressure if the fluctuations of the random forces are given by[12]

$$\langle s_{ij}(\vec{x}, t) s_{lm}(\vec{x}', t') \rangle = 2(k_B T/\rho) \nu \delta(\vec{x} - \vec{x}') \times \delta(t - t')(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) , \quad (8)$$

$$\langle q_i(\vec{x}, t) q_j(\vec{x}', t') \rangle = 2(k_B T^2/C_v) \kappa \delta(x - x') \times \delta(t - t')\delta_{ij} , \quad (9)$$

$$\langle q_i(\vec{x}, t) s_{jk}(\vec{x}', t') \rangle = \langle q_i(\vec{x}, t) \rangle = \langle s_{jk}(\vec{x}, t) \rangle = 0 . \quad (10)$$

We wish to evaluate the fluctuations of hydrodynamic variables in the *nonequilibrium* situation in which R is finite. Following previous work,[3,4] we shall *assume* that Eqs. (8)–(10) are unchanged at finite R, since the departures from equilibrium involve macroscopic disturbances, and these should not affect the fluctuations of the forces q and s, which have their origin in microscopic molecular motion. The system of equations (6)–(10) is now complete, and we may attempt to calculate the correlation functions of the variables T, u, and p for $R \to R_c$. In the case where the equations of motion can be *linearized* in θ and u the calculation is rather straightforward, and was carried out by Zaitsev and Shliomis.[3] The important variables are $u_z(\vec{x}, t)$ and $\theta(\vec{x}, t)$, from which two linear combinations can be formed, with characteristic frequencies

$$\omega_1(\vec{p}) = \frac{\nu(q_0^2 + p_z^2)}{1 + P} \left[\left(\frac{R_c - R}{R_c} \right) + \frac{(q^2 - q_0^2)^2}{q_0^2 (q_0^2 + p_z^2)} \right], \quad (11)$$

$$\omega_2(\vec{p}) = \nu(q_0^2 + p_z^2)(1 + P)/P , \quad (12)$$

respectively, for $R \to R_c$. In Eqs. (11) and (12), whose derivation is outlined in Appendix A, $\vec{p}$ is the three-dimensional wave vector conjugate to $\vec{x}$, and $P \equiv \nu/\kappa$ is the Prandtl number. Writing $p^2 = q^2 + p_z^2$, where $\vec{q}$ is a two-dimensional vector in the horizontal plane, we fix p_z, q_0, and R_c by the boundary conditions on the top and bottom plates.

Let $w(\vec{x}, t)$ be the linear combination of v_z and θ corresponding to the "slow" mode $\omega_1(p)$. Then as shown in Appendix A, the Fourier transform of the correlation function of w is equal to

$$C_{ww}(q, \omega) = \frac{1+P}{(q^2-q_0^2)^2/q_0^2(q_0^2+p_z^2)+(R_c-R)/R_c} \times \left[\frac{k_B T}{\rho} + \frac{P k_B T^2}{C_v}\left(\frac{\nu(q_0^2+p_z^2)}{\Delta T/l}\right)^2\right]\frac{\omega_1}{\omega^2+\omega_1^2} \tag{13}$$

for $q \to q_0$, $R \to R_c$. This correlation function displays the "critical slowing down," and the divergent "static susceptibility" characteristic of a second-order phase transition.[13] Moreover, the exponents are clearly those of the mean-field and conventional theories[7] of critical behavior. The correlation function in Eq. (13) is precisely the one which would be obtained from a "time-dependent Ginzburg-Landau" equation[7] of the form

$$\frac{\partial w(\vec{x}_\perp, t)}{\partial t} = -\Gamma_0 \chi_0 \frac{\delta F}{\delta w(\vec{x}_\perp, t)} + \eta(\vec{x}_\perp, t) , \tag{14}$$

with the Gaussian "free energy"

$$F_G = \frac{1}{2k_B T \chi_0}\int d^2x_\perp \left[w(\vec{x}_\perp)\left(\frac{R_c-R}{R_c} + \frac{(\nabla_\perp^2+q_0^2)^2}{(q_0^2+p_z^2)q_0^2}\right) \times w(\vec{x}_\perp)\right], \tag{15}$$

$$\langle \eta(\vec{x}_\perp, t)\eta(\vec{x}_\perp', t')\rangle = k_B T \Gamma_0 \chi_0 \delta(\vec{x}_\perp - \vec{x}_\perp')\delta(t-t') , \tag{16}$$

and with

$$\Gamma_0 = \nu(q_0^2+p_z^2)(1+P)^{-1} , \tag{17a}$$

$$\chi_0 = (1+P)\left[\frac{1}{\rho} + \left(\frac{\nu(q_0^2+p_z)}{\Delta T/l}\right)^2 \frac{PT}{C_v}\right] p_z . \tag{17b}$$

Nonlinear terms

The preceding analysis holds for R close to R_c, where one can neglect the fast mode varying with the finite frequency $\omega_2(\vec{p})$, Eq. (12). Since the nonlinear terms in Eq. (6) were neglected in the analysis, the results will not hold in the immediate vicinity of the transition, where the linear theory predicts diverging fluctuations. One must therefore include the nonlinear terms of Eq. (6), to the extent that these affect the slow mode (11). As shown in Appendix A, these terms lead to a modification of the free energy, which takes the form

$$F = F_G + \frac{g_0 P}{4!\, k_B T \chi_0 \Gamma_0 \kappa (1+P)^3}\int d^2x_\perp\, w^4(\vec{x}_\perp) , \tag{18}$$

where g_0 is a numerical constant.

The Bénard system near R_c has thus been reduced to the study of a relaxational model given by the general equation of motion (14), with the free energy (15)–(18). This model contains a small parameter $k_B T/\nu^2 l\rho \approx 10^{-9}$ (for $l = 1$ cm), which is the ratio of the noise energy to the characteristic energy in F, i.e., the viscous dissipation associated with one Bénard cell. Thus fluctuation effects will only occur extremely close to R_c. In order to make the small parameter apparent we introduce dimensionless variables

$$\psi = w[\nu(q_0^2+p_z^2)^{1/2}B]^{-1}, \quad \zeta = \frac{\eta(1+P)}{\nu^2(q_0^2+p_z^2)^{3/2}B} ,$$

$$t = \frac{\tilde{t}(1+P)}{\nu(q_0^2+p_z^2)} , \quad \vec{x} = \tilde{\vec{x}}_\perp/q_0 ,$$

where

$$B^2 = k_B T \Gamma_0 \chi_0 \frac{(1+P)^2 q_0^2}{\nu(q_0^2+p_z^2)^2} ,$$

so that our model takes the form

$$\frac{\partial\psi}{\partial\tilde{t}} = \frac{\delta\mathfrak{F}}{\delta\psi} + \zeta(\tilde{\vec{x}}, \tilde{t}) , \tag{19}$$

$$\langle \zeta(\tilde{\vec{x}}, \tilde{t})\zeta(\tilde{\vec{x}}', \tilde{t}')\rangle = \delta(\tilde{\vec{x}} - \tilde{\vec{x}}')\delta(\tilde{t}-\tilde{t}') , \tag{20}$$

$$\mathfrak{F} = \frac{1}{2}\int d^2\tilde{x}\, \psi(\tilde{\vec{x}})\left(\tau + \frac{(\nabla_{\tilde{x}}^2+q_0^2)^2}{q_0^2(q_0^2+p_z^2)}\right)\psi(\tilde{\vec{x}}) + \frac{\lambda}{4!}\int d^2\tilde{x}\, \psi^4(\tilde{\vec{x}}) , \tag{21}$$

where

$$\tau = (R_c - R)/R_c , \tag{22a}$$

and

$$\lambda = g_0 P^2 \frac{k_B T q_0^2}{\nu^2(q_0^2+p_z^2)^2}\left[\frac{1}{P} + \left(\frac{\nu(q_0^2+p_z^2)}{\Delta T/l}\right)^2 \frac{PT}{C_v}\right] p_z \approx 10^{-9} \tag{22b}$$

reflects the smallness of the fluctuation effects. The system defined by Eqs. (14) and (18) is very close to the one studied by Graham,[4,5] with the important difference that the rotational symmetry of the starting equations in the horizontal plane has been preserved in our model.

III. PROPERTIES OF BRAZOVSKII'S MODEL

The free energy (21) is precisely of the type considered by Brazovskii[9] in his study of the condensation of a liquid to a nonuniform state. The phase transition involves a "condensation" from a uniform system $\langle\psi\rangle = 0$ or $\langle w\rangle = 0$, to a periodic array (density wave) characterized by one or more vectors $\vec{q}_i$, such that $|\vec{q}_i| = q_0$. This model differs from the usual model[8] for the condensation of a liquid in that the free energy contains no cubic terms, which would lead to a first-order transition in mean-field theory.[8] The absence of such cubic terms in the Bénard problem is due to the special symmetry of our problem (see Appendix A).

Our system also differs from the usual n-vector model of critical points,[10] in that the order parameter of the latter model has a unique value of the wave vector associated with it in the condensed phase,

$$\langle\psi_\alpha(\vec{q})\rangle = M_\alpha \delta(\vec{q}-\vec{q}_0)\delta_{\alpha,1}\ , \tag{23}$$

where $\vec{q}_0=0$ for a ferromagnet, and $\vec{q}_0$ is any one of a set of isolated points in reciprocal space for a typical antiferromagnet. In the system represented by Eq. (21), the condensation can occur for any $\vec{q}$ on a circle [or, more generally, a $(d-1)$-dimensional sphere $|\vec{q}|=q_0$]. Thus the broken symmetry of the condensed phase involves not just the choice of a particular component $\alpha=1$ of the n-component vector $\psi_\alpha(\vec{q}_0)$, as in (23), but also the choice of a vector $\vec{q}=\vec{q}_1$ out of the *infinite* set of equivalent order parameters $\psi(\vec{q})$ with $|\vec{q}\,|=q_0$. We shall refer to the usual case as a "discrete" condensation, and to the system (21) as a "continuous" condensation. In some sense the system (21) is equivalent to an n-component spin system with $n=\infty$. An important difference, however, is that in the discrete system there is a d-dimensional phase space of $\vec{q}$ values around $\vec{q}_0$, which leads to small energy differences and large fluctuations. In the continuous model, the d-dimensional phase space about $\vec{q}_1$ involves partly fluctuations to points $\vec{q}_i$ on the "sphere" of condensation, which are exactly degenerate ($|\vec{q}_i|=q_0$), and partly points $\vec{q}$ away from this sphere ($|\vec{q}|\neq q_0$), which are only nearly degenerate. Thus the continuous-condensation model does not have the full d-dimensional phase space for fluctuations away from the ordered state. It should be clear from the preceding discussion that the system (21), which was discussed by Brazovskii,[9] does not belong to any of the usual universality classes of isotropic models[10] characterized by parameters n and d. Indeed, this is true of any model describing the spatial ordering (crystalization) of a liquid, but in most cases terms of third order in ψ lead to a first-order transition, and the question of the universality class of the fluctuations does not arise. The Brazovskii model is noteworthy precisely because such terms are absent, and the transition is second order when fluctuations are neglected.

Brazovskii's solution

Brazovskii has studied the properties of the model in Eq. (21) in three dimensions, and has argued that it will undergo a first-order transition, when fluctuations are taken into account.[14] This transition, which occurs below the mean-field value $\tau=0$, is obtained by calculating the difference in free energy between the disordered state and a state with periodic density. The temperature at which this difference changes sign is the first-order transition temperature. Since fluctuations are important in suppressing the second-order transition, it is necessary to take them into account consistently in estimating the free energy, and Brazovskii has attempted to do this, in the weak coupling limit (see Appendix B).

Of the different ordered states which are possible for a scalar order parameter, Brazovskii found that the state with the density varying in only one direction will be most favorable energetically. Since, however, such a state does not have true long-range order in three dimensions,[15] there will be no true difference of symmetry between the states above and below the transition. (Such a difference of symmetry is of course not necessary in a first-order transition.)

In attempting to generalize Brazovskii's work to a two-dimensional system, with condensation along a circle $|\vec{q}\,|=q_0$, one encounters certain additional difficulties having to do with the absence of true long-range order. Nevertheless, for a system of finite but large lateral dimensions we can repeat Brazovskii's argument, to show that there will be a first-order transition to a state with nonuniform density. As in the three-dimensional case, the density varies in one direction only in the lowest state, which corresponds to a roll pattern in the language of convective instabilities.[2] The arguments, which are outlined in Appendix B, are not rigorous, since they are based on a rough estimate of the omitted fluctuation terms. Moreover, they hold only for weak coupling ($\lambda\ll 1$). The jump in the value of $\langle\psi\rangle$ is of order $\lambda^{-1/6}$, so that the jump in the value of $\langle w\rangle$ ($\sim \nu q_0\lambda^{1/3}$) will not be observable, in practice, in the Bénard case, where $\lambda\sim 10^{-9}$.

Effects of finite lateral dimension L

As shown in Appendix B, in the two-dimensional case, the argument of Brazovskii breaks down when the lateral dimension L exceeds $\lambda^{-2/5}q_0^{-1}$. In that case ($L\gg\lambda^{-2/5}q_0^{-1}$) it is not clear whether the transition will be sharp or smeared. For $L\ll\lambda^{-2/5}q_0^{-1}$ there appears to be a sharp transition, as discussed above, until L becomes small enough so that the system is "zero-dimensional." This occurs when the correlation length at the "temperature" of onset of fluctuations ($|\tau|\approx\lambda^{2/3}$) is of order L, namely, for $q_0L\lesssim\lambda^{-1/3}$ (since $\xi\sim|\tau|^{-1/2}$).

We therefore can identify three regimes, depending on the relative size of the lateral dimension L, the coupling constant λ (assumed $\ll 1$), and the wave vector $q_0 \approx l^{-1}$ (where l is the plate separation in the Bénard problem).

(i) $Lq_0 \gg \lambda^{-2/5}$: Brazovskii's analysis does not yield any estimate for the properties of the system in the transition region in two dimensions, since his perturbation series is in powers of $\lambda(q_0L)^{5/2}$.

(ii) $\lambda^{-2/5} \gg Lq_0 \gg \lambda^{-1/3}$: Brazovskii's arguments lead to a first-order transition at a reduced temperature $\tau \approx -\lambda^{2/3}$.

(iii) $\lambda^{-1/3} \gg Lq_0$: When τ becomes small enough so that fluctuations are important the correlation length is already large compared to L, and the transition is smeared by "zero-dimensional" behavior. This is the regime studied by Graham[4] and by Smith,[6] and as they point out, the rounding will only occur very close to $\tau = 0$ (i.e., at $|\tau| \approx \lambda^{2/3}$).

When the coupling constant λ is not very small compared to unity, Brazovskii's treatment does not apply, and it is not clear how the system will behave. It should be remarked that since we are predicting a first-order transition with no strict change in symmetry, there is no reason why the behavior should be universal as a function of the coupling constant.

We have not explicitly studied the time-dependent problem defined in Eqs. (19)–(21). Nevertheless, it is clear that the characteristic frequency of the order parameter will roughly follow the inverse susceptibility r, and will experience a jump, if there is a first-order transition at $\tau = -\tau_c$.

IV. COMPARISON WITH OTHER WORK

The first estimate of the effect of hydrodynamic fluctuations on the Bénard instability was that of Zaitsev and Shliomis,[3] who used the linear theory, and found a sharp "second-order" transition with diverging fluctuations, and "mean-field" exponents. As in any mean-field treatment, they found no rounding effect due to the finite lateral dimensions of their system. As mentioned earlier, such a rounding was then discussed by Graham[4] and by Smith[6] using the nonlinear theory, in the (zero-dimensional) regime where only one mode is excited, as in a laser. The range of values of $R - R_c$ where this effect is significant is of course very small, because of the weakness of the fluctuating forces assumed in the model. We recover the results of Refs. 4 and 6 from our model in the zero-dimensional limit $Lq_0 < \lambda^{-1/3}$.

For systems with large lateral extension one can excite many nearly degenerate modes, and the fluctuation effects will depend sensitively on the symmetries of the system. Fluctuation effects in a large system were studied by Graham[4,5] using a model very similar to ours. The main difference is that Graham chose a particular direction for condensation, and did not preserve the isotropic symmetry of the disordered phase. This means that he did not take into account the full spectrum of fluctuations of the ideal system. The results he found for finite but large systems with lateral dimensions $L_x = L_y$ have an asymmetry between x and y which is unphysical, since the system is assumed to be isotropic, at least for $R < R_c$ ($\tau > 0$). We therefore disagree with the results of Sec. 4.2 of Ref. 5.

An analysis of the infinite system was given by Graham and Pleiner[11] using a mode-coupling formalism applied directly to the hydrodynamic equations. As we explained earlier, we believe that our model is completely equivalent to the hydrodynamic starting point, insofar as one is only interested in singular properties of the system very near the instability. The differences between our model (or that of Ref. 4) and the more-complicated one of Graham and Pleiner[11] have to do with properties which are "irrelevant" in the renormalization-group sense.[10] Although we have not made a detailed comparison between the work of Graham and Pleiner[11] and our own, we believe that the arguments leading to the equation between (50) and (51) of Ref. 11 [GP (50a)] are parallel to the arguments by which we proceed from the hydrodynamic equations to our Hartree equation (B2) of Appendix B. Indeed the v_{ijk} in Eq. (29) of Ref. 11 are analogous to the v in our Eq. (A18).

An important formal difference between our work and Ref. 11 is that we have separated the static and dynamic problems, by first making the simplications of neglecting fast modes and considering a problem involving the slow mode only. Equation (29) of Ref. 11 corresponds to a diagram like that of Fig. 1(a). If the dashed line corresponds to the propagator for a fast mode, then its frequency dependence is not singular at low frequencies. Thus a fast-model propagator may presumably be replaced by a constant[16] in Eq. (29) of Ref. 11, which then becomes a Hartree self-energy [see Eq. (B2) below].

Thus it seems to us that a central result of Graham and Pleiner,[11] their Eq. (50a), is essentially equivalent to our first-order Hartree approximation Eq. (B2). Their equation reads

$$B(\nu)[B(\nu)+\tfrac{3}{2}\pi^2\nu]^{1/2} = B_0^{3/2}(1-\gamma\nu)\ , \qquad (24)$$

where $B(\nu)$ is related to a self-energy by their Eq. (50),

$$\Sigma_1^{(1)}(\nu, \delta=0) = -B(\nu)/(P+1)\ , \qquad (25)$$

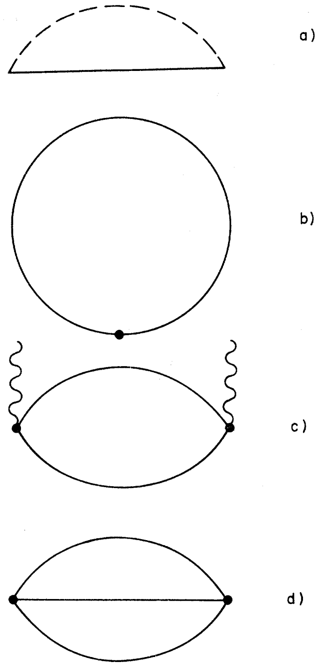

FIG. 1. (a) A diagram which appears in the formalism of Ref. 11. The dashed line represents a propagator for a "fast" mode, and the solid line a propagator for the "slow" mode as discussed in the text. (b) Hartree diagram. The dot represents a factor of λ, while the solid line represents a factor of $g_H(q)=[r_H+(q^2-q_0^2)^2]^{-1}$. (c) A second-order "anomalous" diagram which appears in perturbation theory in the ordered phase. The wavy lines represent factors of $2a\cos\vec{q}_1\cdot\vec{x}_\perp$. (d) A second-order diagram.

with $\nu=(R_c-R)/R_c$. If we identify their B_0 with our $\lambda^{2/3}$ and neglect the $\gamma\nu$ term on the right-hand side of (24), then our Eq. (B2) corresponds to (24) to within numerical factors. Equation (24) leads Graham and Pleiner to predict a smeared transition as in a one-dimensional spherical model.[13] Comparing with the discussion of Appendix B we see that the effects associated with the ordered phase have been left out of the analysis of Ref. 11, which therefore misses the first-order transition. Moreover, the contribution from higher-order terms [for example, the one represented in Fig. 1(d)] will in general not be negligible sufficiently far below $\nu=0$ $(R>R_c)$. In the range $\lambda^{2/5}\ll(Lq_0)^{-1}\ll\lambda^{1/3}$ we have argued that these terms are negligible, but in general they are expected to invalidate the Hartree approximation in Eq. (24).

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APPENDIX A: EQUIVALENCE OF BÉNARD PROBLEM AND BRAZOVSKII'S MODEL

Linear analysis

The linearized versions of Eqs. (6) and (7) for u_z, the z component of the velocity, and θ are

$$\frac{\partial}{\partial t}\nabla^2 u_z-\nu\nabla^2\nabla^2 u_z-\alpha g\nabla_\perp^2\theta=f_z\,, \tag{A1}$$

$$\frac{\partial}{\partial t}\theta-\kappa\nabla^2\theta-\beta u_z=f_\theta\,, \tag{A2}$$

where

$$\beta=\Delta T/l\,,$$

$$f_z=-\vec{\nabla}\times(\vec{\nabla}\times\vec{\nabla}\cdot\vec{s})\,,$$

and

$$f_\theta=-\vec{\nabla}\cdot\vec{q}\,.$$

Solve by Fourier transformation for the correlation function,

$$u_z(\vec{x},t)=\int\frac{d^2q}{(2\pi)^2}\frac{d\omega}{2\pi}\times e^{i(\vec{q}\cdot\vec{x}_\perp-\omega t)}\tilde{u}(\vec{q},\omega)\sin p_z z\,, \tag{A3}$$

$$\theta(\vec{x},t)=\int\frac{d^2q}{(2\pi)^2}\frac{d\omega}{2\pi}\times e^{i(\vec{q}\cdot\vec{x}_\perp-\omega t)}\tilde{\theta}(\vec{q},\omega)\sin p_z z\,, \tag{A4}$$

where we assume free-free boundary conditions at the horizontal plate,[2] and where $p_z=\pi/l$; i.e., we restrict ourselves to the lowest vertical mode. Note that the Fourier integrals (A3) and (A4) are appropriate for a system of infinite lateral extent.

Define (where $p^2=q^2+p_z^2$)

$$L_0=\begin{bmatrix}\nu p^2 & -\alpha g q^2/p^2\\ -\beta & \kappa p^2\end{bmatrix}. \tag{A5}$$

For R near $R_c=657.5$ and $|\vec{q}|$ near q_0 $(q_0^2=p_z^2/2)$, L_0 has slow and fast eigenvalues, ω_1 and ω_2, respectively, given in Eqs. (11) and (12) of the text. Let w be the combination of u_z and θ which corresponds to the left eigenvector of L_0 associated with the slow eigenvalue ω_1,

$$\tilde{w}(q,\omega)=\tilde{u}_z(q,\omega)+(\nu p_0^2/\beta)\tilde{\theta}(q,\omega)\,, \tag{A6}$$

where $p_0^2=p_z^2+q_0^2$. Then, by use of (A2) and (8)–(10), and

$$\langle\tilde{w}(\vec{q},\omega)\tilde{w}(\vec{q}',\omega')\rangle\equiv 2(2\pi)^3\delta(\vec{q}+\vec{q}')\times\delta(\omega+\omega')C_{ww}(\vec{q},\omega)\,, \tag{A7}$$

we find that $C_{ww}(\vec{q}, \omega)$ is given by Eq. (13) of the text.

Nonlinear terms

Write the full nonlinear equations (6) and (7) in a matrix form,

$$\mathcal{L}_0\Psi + v{:}\Psi\Psi = f\ , \tag{A8}$$

where Ψ is a four-component column vector

$$\Psi = \begin{bmatrix} u_z \\ \theta \\ u_x \\ u_y \end{bmatrix} \tag{A9}$$

and where $\mathcal{L}_0$ is the matrix operator

$$\mathcal{L}_0 = \begin{bmatrix} \frac{\partial}{\partial t}\nabla^2 - \nu\nabla^2\nabla^2 & -\alpha g \nabla_\perp^2 & 0 & 0 \\ -\beta & \frac{\partial}{\partial t} - \kappa\nabla^2 & & \\ 0 & \alpha g \nabla_x \nabla_z & \frac{\partial}{\partial t}\nabla^2 - \nu\nabla^2\nabla^2 & 0 \\ 0 & \alpha g \nabla_y \nabla_z & 0 & \frac{\partial}{\partial t}\nabla^2 - \nu\nabla^2\nabla^2 \end{bmatrix}; \tag{A10}$$

f is a column vector composed of the fluctuating forces. The symbol $v{:}\Psi\Psi$ is used to denote the four-component vector

$$v{:}\Psi\Psi = \begin{bmatrix} \nabla_z\nabla_j\nabla_k(u_k u_j) - \nabla_k\nabla_k\nabla_j u_z u_j \\ -\vec{u}\cdot\nabla\theta \\ \nabla_x\nabla_j\nabla_k(u_k u_j) - \nabla_k\nabla_k\nabla_j(u_x u_j) \\ \nabla_y\nabla_j\nabla_k(u_k u_j) - \nabla_k\nabla_k\nabla_j(u_y u_j) \end{bmatrix}, \tag{A11}$$

and represents the nonlinearity in which we are expanding. In (A11), the summation convention is used and the indices j and k range over the Cartesian components x, y, z.

Let us introduce a factor ϵ in front of the nonlinear term in (A8), do second-order perturbation theory in ϵ, and then set $\epsilon = 1$.[17] A small parameter appears later in the theory as discussed in the text. Equation (A8) becomes

$$\mathcal{L}_0\Psi + \epsilon v{:}\Psi\Psi = f\ . \tag{A12}$$

Write

$$\Psi = \Psi^{(0)} + \epsilon\Psi^{(1)} + \epsilon^2\Psi^{(2)}\ . \tag{A13}$$

Then we obtain, from collecting the factors multiplying powers of ϵ through ϵ^2,

$$\mathcal{L}_0\Psi^{(0)} = f\ , \tag{A14}$$

$$\mathcal{L}_0\Psi^{(1)} + v{:}\Psi^{(0)}\Psi^{(0)} = 0\ , \tag{A15}$$

$$\mathcal{L}_0\Psi^{(2)} + v{:}\Psi^{(0)}\Psi^{(1)} + v{:}\Psi^{(1)}\Psi^{(0)} = 0\ . \tag{A16}$$

We may invert Eq. (A15) to find $\Psi^{(1)}$. This is so because $\Psi^0\Psi^0$ contains only the second and zeroth harmonics in p_z and not the first.[18] Hence $\mathcal{L}_0$ operating on the second harmonics $\sin 2p_z z$ and $\cos 2p_z z$ will not be zero, and the inversion can be performed. We also project $\Psi^{(0)}$ in the nonlinear term in (A15) onto the slow and fast eigenvectors discussed above, and keep only the slow part. Thus

$$\Psi^{(1)} = -\mathcal{L}_0^{-1} v{:}\Psi^{(0)}\Psi^{(0)}\ , \tag{A17}$$

and we have

$$\begin{aligned} \mathcal{L}_0\Psi &= \mathcal{L}_0(\Psi^{(0)} + \epsilon\Psi^{(1)} + \epsilon^2\Psi^{(2)}) \\ &= f - \epsilon v{:}\Psi^{(0)}\Psi^{(0)} + \epsilon^2 v{:}\Psi^{(0)}\mathcal{L}_0^{-1} v{:}\Psi^{(0)}\Psi^{(0)} \\ &\quad + \epsilon^2 v{:}(\mathcal{L}_0^{-1} v{:}\Psi^{(0)}\Psi^{(0)})\Psi^{(0)}\ . \end{aligned} \tag{A18}$$

The right-hand side of Eq. (A18) contains response in the zeroth, first, second, and third harmonics of $p_z z$. We retain the response only in the first harmonic. To the order we are working we may replace $\Psi^{(0)}$ by Ψ in the last two terms in Eq. (A18). If we write

$$u_z(\vec{x}, t) = u_z'(\vec{x}_\perp, t)\sin p_z z\ , \tag{A19}$$

$$\theta(\vec{x}, t) = \theta'(\vec{x}_\perp, t)\sin p_z z\ , \tag{A20}$$

then from (A18) we obtain, if we put $\epsilon = 1$,

$$\begin{bmatrix} \frac{\partial}{\partial t}(\nabla_\perp^2 - p_z^2) + \nu(\nabla_\perp^2 - p_z^2)^2 & -\alpha g \nabla_\perp^2 \\ -\beta & \frac{\partial}{\partial t} - \kappa(\nabla_\perp^2 - p_z^2) \end{bmatrix} \begin{bmatrix} u_z'(\vec{x}_\perp, t) \\ \theta'(\vec{x}_\perp, t) \end{bmatrix}$$

$$= \int dz \sin p_z z \begin{bmatrix} f_z(\vec{r}\, t) \\ f_\theta(\vec{r}\, t) \end{bmatrix} - \int \frac{d^2q}{(2\pi)^2} \frac{d^2q'}{(2\pi)^2} \frac{d^2q''}{(2\pi)^2} \frac{g(\vec{q}\vec{q}'\vec{q}'')}{\kappa} \tilde{u}_z(\vec{q}\, t)\tilde{u}_z(\vec{q}', t)\tilde{\theta}(\vec{q}'', t) \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i(\vec{q}+\vec{q}'+\vec{q}'')\cdot\vec{x}}, \quad \text{(A21)}$$

where $g(\vec{q}\vec{q}'\vec{q}'')$ is a complicated function, which may be obtained from (A11). We have calculated g explicitly only in the limit of large Prandtl number, where we find

$$g(\vec{q}\vec{q}'\vec{q}'') = \frac{2}{p_z^4}(1 - \cos\theta_{qq'})(2 + \cos\theta_{qq''} + \cos\theta_{q'q''})[\,|\vec{q}+\vec{q}'|^2 + (2p_z)^2]^2 \frac{1}{h(\vec{q}+\vec{q}')} \quad \text{(A22)}$$

$$h(\vec{q}+\vec{q}') = \left(\frac{|\vec{q}+\vec{q}'|^2}{p_0^2} + 8\right)^2 - \frac{27|\vec{q}+\vec{q}'|^2}{p_0^2}, \quad \text{(A23)}$$

and $\theta_{qq'}$ is the angle between $\vec{q}$ and $\vec{q}'$. Note that the above expression satisfies the condition $g(\vec{q}\vec{q}'\vec{q}'') \geq 0$, but is dependent on the angles between $\vec{q}, \vec{q}', \vec{q}''$.

We take the linear combination of the two equations of (A21), which gives the time derivative of the slow eigenvector of the linear problem, given in Eq. (A6). As mentioned above, we keep only the projections of u_z and θ onto the slow eigenvector. The nonlinear equation for this eigenvector is then

$$\frac{\partial w(\vec{x}_\perp, t)}{\partial t} + \nu p_0^2 \left(\frac{R_c - R}{R_c} + \frac{(\nabla_\perp^2 + q_0^2)^2}{p_0^2 q_0^2}\right)(1+P)^{-1} w(\vec{x}_\perp, t)$$

$$= -\frac{P}{\kappa(1+P)^3} \int \frac{d^2q\, d^2q'\, d^2q''}{(2\pi)^6} \{g(\vec{q}\vec{q}'\vec{q}'')\tilde{w}(\vec{q}, t)\tilde{w}(\vec{q}', t)\tilde{w}(\vec{q}'', t)\exp[i(\vec{q}+\vec{q}'+\vec{q}'')\cdot\vec{x}_\perp]\} + \eta(\vec{x}_\perp, t), \quad \text{(A24)}$$

where

$$\eta(\vec{x}_\perp, t) = 2\int dz \sin p_z z \left(-\frac{1}{p_0^2} f_z(\vec{r}, t) + \frac{\nu p_0^2}{\beta} f_\theta(\vec{r}, t)\right). \quad \text{(A25)}$$

In order to obtain Eq. (18) of the text, it is now necessary to neglect the $\vec{q}$ dependences in the interaction function $g(\vec{q}\vec{q}'\vec{q}'')$. This is in the spirit of the simplest renormalization-group treatment of field models,[10] where a constant interaction vertex is assumed. In the present case, we are also neglecting the angular dependence of g, which may be "relevant" for the critical behavior. We believe that this approximation is justified in the weak-coupling limit, where the system has a first-order phase transition, but a more detailed investigation of the angular dependence of g would be necessary in the strong-fluctuation regime.

If we assume $g(\vec{q}\vec{q}'\vec{q}'')$ to be a constant $\equiv g_0/3!$, we may write (A24) in the form

$$\frac{\partial w}{\partial t} = -\Gamma_0 \chi_0 \frac{\delta F}{\delta w} + \eta, \quad \text{(A26)}$$

where F is given by Eq. (18) in the text.

APPENDIX B: ANALYSIS OF BRAZOVSKII'S MODEL

Let us briefly review Brazovskii's discussion of the ordering in the system described by the free energy (21) in d dimensions, with a scalar order parameter ($n=1$), which we write as

$$\mathfrak{F} = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d} \psi(\vec{q})\psi(-\vec{q})[\tau + (q - q_0)^2] + \frac{\lambda}{4!}\int \frac{d^dq_1}{(2\pi)^d}\int \frac{d^dq_2}{(2\pi)^d}\int \frac{d^dq_3}{(2\pi)^d} \psi(\vec{q}_1)\psi(\vec{q}_2)\psi(\vec{q}_3) \times \psi(-\vec{q}_1 - \vec{q}_2 - \vec{q}_3). \quad \text{(B1)}$$

To relate (21) to (B1), we note that $(q^2 - q_0^2)^2 = (q - q_0)^2(q + q_0)^2 \approx (2q_0)^2(q - q_0)^2$ for the q's of interest. We have also ignored some nonessential constants and have written the free energy in terms of Fourier transforms. The correlation function for the field ψ is

$$g(\vec{x}) = \langle\psi(\vec{x})\psi(0)\rangle,$$

and its Fourier transform $g(q)$ is related to the susceptibility in the usual way. In the disordered phase ($\tau > 0$) the inverse susceptibility

$$r \equiv g^{-1}(|\vec{q}| = q_0) = \tau - \Sigma(|\vec{q}| = q_0)$$

may be expanded in powers of the interaction according to the diagrams of Fig. 1. The first, or "Hartree" term [Fig. 1(b)] yields, in d dimensions,

$$r_H = \tau - \Sigma_H = \tau + \alpha\lambda r_H^{-1/2}\,, \tag{B2}$$

where $\alpha = \pi S_d q_0^{d-1}/(2\pi)^d$, with S_d the surface area of the unit sphere in d dimensions.

It may be noticed that Eq. (B2) is precisely the result obtained for the one-dimensional spherical model[13] ($d=1, n=\infty$) in a discrete system; i.e., different points on the condensation sphere act like different components of the order parameter, and the single degree of freedom perpendicular to this sphere yields the fluctuations of a one-dimensional system. This result is obtained regardless of the dimensionality d of the starting system (B1).

The higher-order terms, given in Fig. 1(d), are of the form

$$\Sigma_d \simeq (\alpha\lambda)^2/q_0 r_H^{3/2}\,, \tag{B3}$$

in any dimension d, if one uses Hartree propagators in the intermediate states. The neglect of Σ_d and higher diagrams in calculating the properties of the disordered phase is justified as long as

$$r_H \gg (\alpha\lambda)^2/q_0 r_H^{3/2}\,, \tag{B4}$$

i.e., for

$$|\tau| \ll (\alpha\lambda)^{3/5} q_0^{1/5}\,. \tag{B5}$$

Note that according (B2) we have τ negative for small r_H.

Similarly, one may use the Hartree self-consistent-field method to estimate the properties of an ordered phase.[9] Let us assume a solution varying in only one direction,

$$\langle\psi(\vec{x})\rangle = 2a\cos\vec{q}_1\cdot\vec{x}\,, \tag{B6}$$

with $|\vec{q}_1| = q_0$. Then one may estimate the free energy of such a solution, and compare it to that of the disordered phase. The difference was evaluated by Brazovskii[9] to be

$$\Delta\Phi = -\frac{r_+^2}{2\lambda} - \frac{\alpha}{2}r_+^{1/2} - \frac{r_-^2}{2\lambda} + \frac{\alpha}{2}r_-^{1/2}\,, \tag{B7}$$

where r_+ is the inverse susceptibility in the Hartree approximation for the disordered phase (B2), and r_- is the inverse susceptibility of the ordered phase in the Hartree approximation, given by

$$r_- = \tau + \alpha\lambda/r_-^{1/2} + \lambda a^2\,. \tag{B8}$$

It may then be verified that for $\tau = -\tau_c' \approx -(\alpha\lambda)^{2/3}$, the free-energy difference $\Delta\Phi$ will change sign. Note that $\tau_c' \approx (\alpha\lambda)^{2/3}$ is indeed smaller than the right-hand side of (B5), so the Hartree approximation is valid.

A difficulty with this argument is that the Hartree approximation is inaccurate at very long wavelengths in the ordered phase, since it does not yield the infinite susceptibility which must be present by symmetry.[19,20] Brazovskii argues that the error involved will be negligible, by assuming the system to be finite and estimating the contribution to the free energy from the omitted terms. These turn out to depend only logarithmically on L in three dimensions, and therefore should not contribute for sufficiently weak coupling.

In two dimensions we may repeat the argument. Consider first the diagram (b) shown in Fig. 1, in the ordered phase. The anomalous contribution of this diagram comes when the argument of the intermediate propagator is near $\vec{q}_1$. This anomalous contribution is

$$\delta\Sigma_b(q\approx q_1) = -\lambda\int\frac{d^2k}{(2\pi)^2}\,\frac{1}{|\vec{q}_1\cdot\vec{k}|^2 + (k^2/2)^2}\,, \tag{B9}$$

where $\vec{k} = \vec{q} - \vec{q}_1$ is a two-dimensional wave vector over which we integrate. Choose a coordinate system in which $\vec{q}_1$ lies parallel to the k_x axis. For a finite system of lateral dimension L, we are then integrating over the k-space region shown in Fig. 2. Note that

$$\begin{aligned}|\vec{q}_1\cdot\vec{k}|^2 + (\tfrac{1}{2}k^2)^2 &= k_x{}^2 q_0^2 + [\tfrac{1}{2}k_x^2 + k_y^2]^2 \\ &= k_x^2(q_0^2 + \tfrac{1}{2}k_y^2 + \tfrac{1}{4}k_x^2) + \tfrac{1}{4}k_y^4 \\ &\approx k_x^2 q_0^2 + \tfrac{1}{4}k_y^4\,.\end{aligned}$$

Therefore,

$$\delta\Sigma_b \approx -\lambda\int_{L^{-1}}^{\Lambda_x}\frac{dk_x}{2\pi}\int_0^{\Lambda_y}\frac{dk_y}{2\pi}\,\frac{1}{k_x^2 q_0^2 + \frac{1}{4}k_y^4}\,, \tag{B10}$$

where Λ_x, Λ_y are upper momentum cutoffs.

The integral converges at the upper limits of integration, so we extend these limits to infinity and find

$$\delta\Sigma_b \approx c_0\lambda(q_0 L)^{1/2}\,, \tag{B11}$$

where c_0 is a numerical constant of order unity.

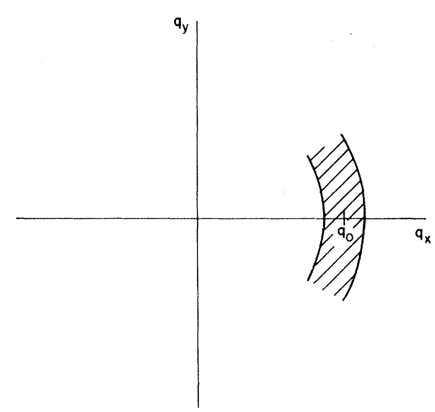

FIG. 2. The region of two-dimensional wave-vector space being integrated over. We exclude the shaded annular region of width L^{-1} about the circle of condensation $|\vec{q}| = q_0$.

Similar considerations lead to a logarithmic dependence of $\delta\Sigma$ on L in three dimensions, in accord with Brasovskii.[9]

Consider next the diagram (c) of Fig. 1. The anomalous contribution $\delta\Sigma_c$ of this diagram may be calculated as above and is

$$\delta\Sigma_c \approx c_1\lambda^2 a^2 (q_0 L)^{5/2}, \tag{B12}$$

where c_1 is also a constant of order unity. Let us finally consider the anomalous contribution $\delta\Sigma_d$ from diagram 1(d). We find

$$\delta\Sigma_d \approx c_d\lambda^2 \int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{1}{k_{1x}^2 q_0^2 + k_{1y}^4}\frac{1}{k_{2x}^2 q_0^2 + k_{1y}^4} \times \frac{1}{(k_{1x}+k_{2x})^2 q_0^2 + (k_{1y}+k_{2y})^4}. \tag{B13}$$

The minimum value for k_{1x} or k_{2x} is L^{-1}. The maximum consistent with the conditions $k_{1y}^2 \lesssim k_{1x}q_0$ and $k_{2y}^2 \lesssim k_{2x}q_0$, etc., is $k_{1y} \lesssim (q_0 L^{-1})^{1/2}$, etc. We therefore estimate that the part of the integral (B13) which diverges most strongly as $L\to\infty$ is

$$\begin{aligned}\delta\Sigma_d &\approx c_d\lambda^2 (q_0L)^{-1}(q_0L)^{-1}(q_0L)^{-1/2} \times (q_0L)^{-1/2}(q_0L^{-1})^{-6}\\ &= c_d\lambda^2(q_0L)^3 = c_d\lambda(q_0L)^{1/2}\lambda(q_0L)^{5/2}.\end{aligned} \tag{B14}$$

In order to be able to neglect (B11) we require

$$\lambda(q_0L)^{1/2} \ll \lambda/r_-^{1/2}, \tag{B15}$$

since $\lambda/r_-^{1/2}$ is a term occurring in Σ which we kept [in (B8)]. The minimum value of r_- is of order $\lambda^{2/3}$; hence we require

$$q_0 L \ll \lambda^{-2/3}. \tag{B16}$$

We also neglect (B12); hence we require

$$\lambda a^2 c_1 \lambda (q_0L)^{5/2} \ll \lambda a^2,$$

as λa^2 is a term which occurs in Eq. (B8). We can satisfy the above inequality if

$$q_0 L \ll \lambda^{-2/5}. \tag{B17}$$

In that case the inequality of Eq. (B16) is satisfied and also the term coming from Eq. (B14) is negligible compared to that from Eq. (B11), which itself may be neglected.

The above argument is the generalization of Brazovskii's discussion to two dimensions, and indicates that in the range $\lambda^{2/5} \ll (Lq_0)^{-1} \ll \lambda^{1/3}$, the effect of symmetry-restoring long-wavelength fluctuations will be negligible and the transition will be first order. Needless to say, this analysis is only suggestive, and could easily be invalidated by an infinite sum of higher-order terms. For example, it may be shown by an argument analogous to the one leading to (B11), that the effect of higher-order graphs is to add to the right-hand side of (B11) terms of the form

$$-c_0\lambda(q_0L)^{1/2}[1 + c_1\lambda L^{5/2} + c_2(\lambda L^{5/2})^2 + \cdots], \tag{B18}$$

where the c_i are numerical constants. The series is a sum of ascending powers of $\lambda L^{5/2}$, which we have assumed to be small, but we have no information on the c_i or the convergence of the series.

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